# LINEAR DIELECTRICS

- Linear dielectrics polarize in proportion to the electric field.

- Expose them to an electric field, they polarize, this changes the field a bit, their polarization changes a bit in response, etc. When things settle down, we have

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

- So the electric displacement for a linear dielectric is:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1 + \chi_e) \vec{E}$$

- Thus,

$$\vec{\nabla} \cdot \vec{D} = \rho_f = \varepsilon_0 (1+\chi_e) \vec{\nabla} \cdot \vec{E} = \cancel{\varepsilon_0}(1+\chi_e)\frac{1}{\cancel{\varepsilon_0}}\rho$$

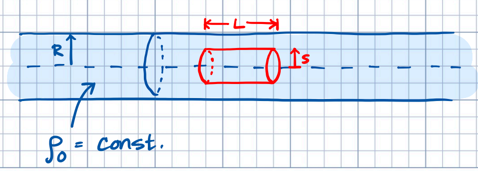
$$\rightarrow \rho_f = (1+\chi_e)\rho = (1+\chi_e)(\rho_b + \rho_f)$$

$$\Rightarrow \rho_b = -\frac{\chi_e}{1+\chi_e}\rho_f$$

Sign is opposite $\rho_f$. For $\chi_e \gg 1$, $\rho_b \simeq -\rho_f$ & $\rho_{TOT} \simeq 0$.

- Since there's a simple rel'n b/t $\vec{D}$ & $\vec{E}$ for a linear dielectric, the Gauss's Law for $\vec{D}$ quickly determines $\vec{E}$ for us.

- EXAMPLE: A long ($\sim \infty$-long) linear dielectric cylinder w/ a constant <u>free</u> charge density inside.




- Gaussian Surface: Cylinder w/ radius $s$ & length $L$
- Cylindrical Symmetry: $\vec{D} = D(s)\hat{s}$

GAUSS: $\oint_{G.S.} d\vec{a}\cdot\vec{D} = q_{f,enc}$ ← Just the $\rho_0$ we introduced, not the $\rho_b$ or $\sigma_b$ from polarization.

$$\hookrightarrow \; 2\pi s L\, D(s) = \begin{cases} \pi s^2 L \rho_0 \,, & s \le R \\ \pi R^2 L \rho_0 \,, & s \ge R \end{cases}$$

$$\Rightarrow \vec{D} = \begin{cases} \frac{s\rho_0}{2}\hat{s} \,, & s \le R \\ \frac{R^2\rho_0}{2s}\hat{s} \,, & s \ge R \end{cases}$$

← $\vec{D}$ cont. @ $s=R$ b/c no $\sigma_f$ on surface of cylinder.

LINEAR DIELECTRIC: $\vec{D} = \varepsilon_0(1+\chi_e)\vec{E}$

$$\Rightarrow \vec{E} = \begin{cases} \frac{s\rho_0}{2\varepsilon_0}\frac{1}{1+\chi_e}\hat{s} \,, & s < R \\ \frac{R^2\rho_0}{2\varepsilon_0 s}\hat{s} \,, & s > R \end{cases}$$

← $\vec{E}$ discontinuous @ $s=R$ b/c polarization gives bound $\sigma_b$ on surface.

Once we know $\vec{D}$, we immediately get $\vec{E}$: we just divide $\vec{D}$ by $\varepsilon_0(1+\chi_e)$. Outside the cylinder there's no dielectric, so $\chi_e = 0$ there.

As a check, look @ the discontinuity in $\vec{E}$ due to $\sigma_b$:

$$\vec{E}_{out}(s=R) - \vec{E}_{in}(s=R) = \frac{R^2\rho_0}{2\varepsilon_0 R}\hat{s} - \frac{R\rho_0}{2\varepsilon_0}\frac{1}{1+\chi_e}\hat{s}$$

$$= \frac{R\rho_0}{2\varepsilon_0}\times\left(1 - \frac{1}{1+\chi_e}\right)\hat{s}$$

$$= \frac{R\rho_0}{2\varepsilon_0}\times\frac{\chi_e}{1+\chi_e}\hat{s}$$

$$\Rightarrow \sigma = \frac{R\rho_0}{2}\frac{\chi_e}{1+\chi_e}$$

This is all bound charge: ← $\varepsilon_0\chi_e\vec{E}$

$$\sigma_b = \hat{n}\cdot\vec{P}\big|_{s=R} = \hat{s}\cdot\left(\frac{R\rho_0}{2}\frac{\chi_e}{1+\chi_e}\hat{s}\right) = \frac{R\rho_0}{2}\frac{\chi_e}{1+\chi_e} \checkmark$$